

Viscous modified gravity on a RS brane embedded in AdS₅

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Abstract We consider a modified gravity fluid on a Randall–Sundrum II brane situated at $y = 0$, the action containing a power α of the scalar curvature. As is known from 4D spatially flat modified gravity, the presence of bulk viscosity may drive the cosmic fluid into the phantom region ($w < -1$) and thereafter inevitably into the Big Rip singularity, even if it is initially nonviscous and lies in the quintessence region ($w > -1$). The condition for this to occur is that the bulk viscosity contains the power $(2\alpha - 1)$ of the scalar expansion. We combine this with the 5D RS II model, and we find that the Big Rip, occurring for $\alpha > 1/2$, carries over to the metric for the bulk metric, $|y| > 0$. Actually, the scale factors on the brane and in the bulk become simply proportional to each other.

1 Introduction

Modified gravity theories in 4D continue to attract interest; this obviously is related to observation, for instance the measured redshifts from type Ia supernovae [1–3]. The data may be reconciled with the concept of dark energy, with a cosmic fluid with a complicated equation of state, or with a scalar field having quintessence or phantom behavior. An extensive recent review is given by Copeland et al. [4]. The equation of state for the cosmic fluid is conventionally written as $p = w\rho$, where $w = -1$ corresponds to a vacuum fluid (cosmological constant), $-1 < w < -1/3$ to a quintessence fluid, and $w < -1$ to a phantom fluid, having the bizarre property of predicting a Big Rip singularity in the future.

In the present paper we combine essentially two kinds of theories.

- (1) First, we assume the modified 4D action in the form given by (18). The integrand contains a power α in the scalar curvature. This model has recently been studied by Abdalla et al. [5]. As a generalization, we include

a bulk viscosity in the fluid that is proportional to the $(2\alpha - 1)$ th power of the scalar expansion; cf. (31). This viscous model has been studied repeatedly in the recent past [6–9]. A striking property of this kind of theory is that it leads to the Big Rip singularity if $\alpha > 1/2$.

- (2) The next step in our analysis is to combine the above 4D theory with the 5D Randall–Sundrum II model [10], where there is a spatially flat brane situated at $y = 0$, surrounded by an AdS space. Our main result is to show how the mentioned Big Rip singularity on the brane becomes transferred to the bulk: the scale factor away from the brane ($|y| > 0$) becomes actually proportional to the scale factor on the brane itself. In this sense the 4D and 5D gravity theories are closely intertwined. In this respect, there is no essential difference between modified gravity and Einstein gravity. They behave qualitatively in the same way, only with the characteristic difference that the strength of the singularity becomes larger for increasing values of α .

2 Basic formalism

Assume, as mentioned, that there is a spatially flat ($k = 0$) brane located at the fifth dimension $y = 0$, surrounded by an anti-de Sitter (AdS) space. If the five-dimensional cosmological constant $\Lambda (< 0)$ is different from zero, this model is the Randall–Sundrum II model (RSII) [10]. We shall take the metric to have the form

$$ds^2 = -n^2(t, y) dt^2 + a^2(t, y) \delta_{ij} dx^i dx^j + dy^2, \quad (1)$$

where $n(t, y)$ and $a(t, y)$ are determined by Einstein's equations:

$$R_{AB} - \frac{1}{2} g_{AB} R + g_{AB} \Lambda = \kappa^2 T_{AB}. \quad (2)$$

The coordinate indices are denoted $x^A = (t, x^1, x^2, x^3, y)$, with $\kappa^2 = 8\pi G_5$ the five-dimensional gravitational coupling. Einstein's equations in a coordinate basis with the

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metric (1) have been given before (cf., for instance, Refs. [11–16], but for convenience we give them also here:

$$3 \left\{ \left(\frac{\dot{a}}{a} \right)^2 - n^2 \left[\frac{a''}{a} + \left(\frac{a'}{a} \right)^2 \right] \right\} - \Lambda n^2 = \kappa^2 T_{tt}, \tag{3}$$

$$a^2 \delta_{ij} \left\{ \frac{a'}{a} \left(\frac{a'}{a} + \frac{2n'}{n} \right) + \frac{2a''}{a} + \frac{n''}{n} + \frac{1}{n^2} \left[\dot{a} \left(-\frac{\dot{a}}{a} + \frac{2\dot{n}}{n} \right) - \frac{2\ddot{a}}{a} \right] + \Lambda \right\} = \kappa^2 T_{ij}, \tag{4}$$

$$3 \left(\frac{\dot{a} n'}{a n} - \frac{\dot{a}'}{a} \right) = \kappa^2 T_{ty}, \tag{5}$$

$$3 \left\{ \frac{a'}{a} \left(\frac{a'}{a} + \frac{n'}{n} \right) - \frac{1}{n^2} \left[\dot{a} \left(\frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) + \frac{\ddot{a}}{a} \right] \right\} + \Lambda = \kappa^2 T_{yy}. \tag{6}$$

Overdots and primes mean derivatives with respect to t and y , respectively. As the $5D$ space outside the brane is taken to be empty, the components of T_{AB} are different from zero only on the brane.

Consider next the form of T_{AB} . Let $U^\mu = (U^0, U^i)$ (Greek indices $\mu, \nu \in [0, 3]$) be the fluid’s four-velocity on the brane, and let σ denote the brane tension, assumed constant. Moreover, let $h_{\mu\nu} = g_{\mu\nu} + U_\mu U_\nu$ be the projection tensor, and $\tilde{p} = p - 3H_0\zeta$ the effective pressure with $H_0 = \dot{a}_0/a_0$ the Hubble parameter on the brane and ζ the bulk viscosity. The shear viscosity is omitted due to the assumed spatial isotropy.

It might be noted here that a viscous fluid may be considered as a specific example of an inhomogeneous equation of state fluid as introduced in Refs. [17, 18].

As a gauge condition we take $n_0(t) = 1$, this meaning that the proper time on the brane is the same as the cosmological time. The energy-momentum tensor can accordingly be written as

$$T_{AB} = \delta(y) (-\sigma g_{\mu\nu} + \rho U_\mu U_\nu + \tilde{p} h_{\mu\nu}) \delta_A^\mu \delta_B^\nu. \tag{7}$$

We shall work in the orthonormal frame where $U^\mu = (1, 0, 0, 0)$, and we let a subscript zero refer to the brane.

The junction conditions at $y = 0$ have now to be taken into account. They express that the metric is continuous across the brane, but its derivatives are not. From (3) and (4) we get, for the distributional parts,

$$\frac{[a']}{a_0} = -\frac{1}{3} \kappa^2 (\sigma + \rho), \tag{8}$$

$$[n'] = \frac{1}{3} \kappa^2 (-\sigma + 2\rho + 3\tilde{p}), \tag{9}$$

where $[a'] = a'(0^+) - a'(0^-)$, and similarly for $[n']$. For the nondistributional parts we get

$$\left(\frac{\dot{a}}{na} \right)^2 - \frac{a''}{a} - \left(\frac{a'}{a} \right)^2 = \frac{1}{3} \Lambda, \tag{10}$$

$$\frac{a'}{a} \left(\frac{a'}{a} + \frac{2n'}{n} \right) + \frac{2a''}{a} + \frac{n''}{n} + \frac{1}{n^2} \left[\dot{a} \left(-\frac{\dot{a}}{a} + \frac{2\dot{n}}{n} \right) - \frac{2\ddot{a}}{a} \right] = -\Lambda. \tag{11}$$

Assuming no energy flux to occur from the brane, we have $T_{ty} = 0$. It implies that

$$n(t, y) = \frac{\dot{a}(t, y)}{\dot{a}_0(t)} \tag{12}$$

for arbitrary y . Then from (10) we get, upon integration with respect to y ,

$$\left(\frac{\dot{a}}{na} \right)^2 = \frac{1}{6} \Lambda + \left(\frac{a'}{a} \right)^2 + \frac{C}{a^4}. \tag{13}$$

Here $C = C(t)$ is an integration constant with respect to y . The C term is called the “radiation term”; as it is not of main interest here, it will be omitted in the following.

Now setting $y = 0$ we get for $H_0 = \dot{a}_0/a_0$

$$H_0^2 = \frac{1}{6} \Lambda + \frac{\kappa^4}{36} (\sigma + \rho)^2. \tag{14}$$

Recall that Λ and σ are constants, while $\rho = \rho(t)$. The generalized Friedmann equation (14) can be contrasted with the conventional Friedmann equation in four-dimensional space (still with $k = 0$),

$$H_0^2 = \frac{1}{3} \Lambda_4 + \frac{1}{3} \kappa_4^2 \rho, \tag{15}$$

with $\kappa_4^2 = 8\pi G_4$. The essential new feature of (14) is thus the occurrence of a ρ^2 term.

Let us observe the solution for $a_0(t)$ from (14) if $\rho = 0$:

$$a_0(t)|_{\rho=0} = \frac{1}{2\sqrt{\lambda}} \exp[\sqrt{\lambda}(t + c_0)], \tag{16}$$

where

$$\lambda = \frac{1}{6} \Lambda + \frac{1}{36} \kappa^4 \sigma^2, \tag{17}$$

c_0 being a new integration constant. The scale factor is thus exponentially increasing, qualitatively as in the ordinary de Sitter case.

3 Modified gravity on the brane

In this section we consider the fluid—Einstein or modified fluid—on the brane $y = 0$. We shall derive how the Hubble parameter H varies with time t , leading eventually to the Big Rip.

We adopt the following 4D gravity model:

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} (f_0 R^\alpha + L_m), \tag{18}$$

where f_0 and α are constants (α may in principle be negative), and L_m is the matter Lagrangian. This model has been studied before; cf., for instance, Refs. [5, 7, 8]. The case $f_0 = 1$ and $\alpha = 1$ corresponds to Einstein gravity. (More complicated $f(R)$ theories have been discussed in various references, for instance in Refs. [17–19]. We may also mention here that a general review of modified gravity can be found in Ref. [20], and recent reviews of viable $f(R)$ gravity theories unifying dark energy, inflation, and dark matter, can be found in Refs. [21, 22].)

The equations of motion following from the action (18) are

$$-\frac{1}{2} f_0 g_{\mu\nu} R^\alpha + \alpha f_0 R_{\mu\nu} R^{\alpha-1} - \alpha f_0 \nabla_\mu \nabla_\nu R^{\alpha-1} + \alpha f_0 g_{\mu\nu} \nabla^2 R^{\alpha-1} = \kappa_4^2 T_{\mu\nu}, \tag{19}$$

where $T_{\mu\nu}$ corresponds to the term L_m in the Lagrangian. (The proposal of taking $f(R)$ on the brane was considered also in Ref. [23].)

We take the equation of state for the fluid to have the conventional form:

$$p = w\rho \equiv (\gamma - 1)\rho \tag{20}$$

(more complicated forms for the equation of state have recently been investigated by [17, 18, 24]). If $w = -1$ or $p = -\rho$, we have a “vacuum fluid”, with bizarre thermodynamic properties such as possibly negative entropies [25]. As is known, cosmological observations indicate that the present universe is accelerating. Moreover, based upon the observed data it has been conjectured that w is a varying function of time. For instance, as discussed in Ref. [26], w might have been around 0 at redshift $z \sim 1$ and may be slightly less than -1 today. Perhaps w is even an oscillating function in time. Recent discussions on possible forms of the equation of state are given, for instance, in Refs. [27, 28]. In view of these circumstances the analysis of a possible crossing of the phantom barrier, $w = -1$, from the quintessence region ($-1 < w < -1/3$) into the phantom region $w < -1$, is obviously of physical interest. It should be noted that both quintessence and phantom fluids lead to the inequality $\rho + 3p \leq 0$, thus breaking the strong energy condition. The Big Rip singularity has been discussed in various papers; cf., for instance, Refs. [29–33]. Specifically, the future singularities in modified gravity were considered also in Ref. [34].

Of main interest is the (00)-component of (19). Observing that $R_{00} = -3\ddot{a}/a$ and $R = 6(\dot{H} + 2H^2)$, as well as

$T_{00} = \rho$, we obtain

$$\frac{1}{2} f_0 R^\alpha - 3\alpha f_0 (\dot{H} + H^2) R^{\alpha-1} + 3\alpha(\alpha - 1) f_0 H R^{\alpha-2} \dot{R} = \kappa_4^2 \rho. \tag{21}$$

An important property of (21) is that the covariant divergence of the LHS is equal to zero [35],

$$\nabla^\nu T_{\mu\nu} = 0, \tag{22}$$

just as in Einstein’s gravity. Energy-momentum conservation is a consequence of the field equations. This leads to the energy conservation equation

$$\dot{\rho} + (\rho + p)3H = 9\zeta H^2. \tag{23}$$

We now differentiate the expression (21) with respect to t , and insert $\dot{\rho}$ from (23). After some calculation we obtain

$$\frac{3}{2} \gamma f_0 R^\alpha + 3\alpha f_0 [2\dot{H} - 3\gamma(\dot{H} + H^2)] R^{\alpha-1} + 3\alpha(\alpha - 1) f_0 [(3\gamma - 1)H\dot{R} + \ddot{R}] R^{\alpha-2} + 3\alpha(\alpha - 1)(\alpha - 2) f_0 \dot{R}^2 R^{\alpha-3} = 9\kappa_4^2 \zeta H. \tag{24}$$

Recalling that $R = 6(\dot{H} + 2H^2)$, we see that this equation is a complicated nonlinear differential equation for $H(t)$. It is best discussed in terms of examples. We are interested in solutions that are related to the Big Rip. We shall look for solutions having the form

$$H = \frac{H_*}{X}, \quad \text{where } X \equiv 1 - BH_*t. \tag{25}$$

Here H_* is the Hubble parameter at the present time, $t = 0$ (usually called H_0 , but we are reserving the subscript zero mainly for brane entities), and B is a nondimensional constant. For the Big Rip to occur, B has to be positive.

3.1 Einstein’s gravity fluid

As mentioned above, this case corresponds to $f_0 = 1$ and $\alpha = 1$. As for the bulk viscosity, we shall take ζ to be proportional to the scalar expansion $\theta = 3H$ through a proportionality constant, here called τ_E ,

$$\zeta = \tau_E \theta = 3\tau_E H. \tag{26}$$

This form is of particular physical interest. Namely, as shown in Ref. [6], if τ_E is large enough to satisfy the condition

$$\chi \equiv -\gamma + 3\kappa_4^2 \tau_E > 0, \tag{27}$$

then the equations of motion lead to the Big Rip singularity in a finite time t . Even if one starts with a state in which

the fluid is nonviscous and lies in the quintessence region ($\gamma > 0$), the imposition of a sufficiently large bulk viscosity will drive it into the phantom region and thereafter inevitably into the Big Rip.

From the governing equations we now get

$$B = \frac{3}{2}\chi, \tag{28}$$

$$H_* = \sqrt{\frac{1}{3}\kappa_4^2 \rho_*}, \tag{29}$$

$$\rho_E = \frac{\rho_*}{X^2}, \tag{30}$$

where ρ_* is the $t = 0$ value of the energy density.

3.2 Modified gravity fluid

Assume now that f_0 and α are arbitrary. Let the bulk viscosity for the modified fluid be denoted by ζ_α . As in Refs. [8, 9] we model ζ_α by setting it proportional to the $(2\alpha - 1)$ th power of the scalar expansion:

$$\zeta_\alpha = \tau_\alpha \theta^{2\alpha-1} = \tau_\alpha (3H)^{2\alpha-1}. \tag{31}$$

The main reason for this assumption is that it fits well with the governing equation for H as well as with our previous assumption (26): the time-dependent factors in (24) automatically drop out, and we are left with the following equation determining B :

$$\begin{aligned} & (B + 2)^{\alpha-1} \{9(2 - \alpha)\gamma \\ & + 3[\alpha + 3\gamma + \alpha(2\alpha - 3)(3\gamma - 1)]B \\ & + 6\alpha(\alpha - 1)(2\alpha - 1)B^2\} = \frac{18\kappa_4^2}{f_0} \left(\frac{3}{2}\right)^\alpha \tau_\alpha. \end{aligned} \tag{32}$$

This equation is complicated and is best discussed in terms of examples. For instance, if $\alpha = 2$ and $\gamma = 0$ (the latter condition corresponding to a vacuum fluid), then (32) yields the following cubic equation ($\tau_\alpha \rightarrow \tau_2$):

$$B^3 + 2B^2 = \frac{9\kappa_4^2 \tau_2}{8f_0}. \tag{33}$$

There exists one single positive root for this equation, as long as the RHS is positive. This root is caused by viscosity and leads to the Big Rip.

We note also the general equation for B following directly from the energy conservation equation (23) for the modified fluid,

$$\dot{\rho}_\alpha + (\rho_\alpha + p_\alpha)3H = 9\zeta_\alpha H^2, \tag{34}$$

namely

$$B = -\frac{3\gamma}{2\alpha} + \frac{3\tau_\alpha}{2\alpha} \frac{(3H_*)^{2\alpha}}{\rho_*}, \tag{35}$$

where we used

$$\zeta_\alpha = \tau_\alpha \left(\frac{3H_*}{X}\right)^{2\alpha-1}, \quad \rho_\alpha = \frac{\rho_*}{X^{2\alpha}}. \tag{36}$$

For simplicity we have assumed the same initial conditions at $t = 0$ for the modified fluid as for the Einstein fluid, viz. $\rho_{*\alpha} = \rho_{*E} \equiv \rho_*$ and $H_{*\alpha} = H_{*E} \equiv H_*$.

4 Implications for the 5D theory

We are now equipped with the necessary background to see how the modified fluid on the brane affects the 5D brane physics. Consider first (14) on the brane (recall that this is a 5D, not a 4D equation). It is natural from a physical point of view to use the expressions for $\rho(t)$ from the previous section as input quantities in this equation. Comparison of (36) and (30) shows that we can regard $\rho_\alpha = \rho_*/X^{2\alpha}$ as a generic equation common to the two cases, only with $\alpha = 1$ in the Einstein case. For the 5D scale factor $a_0(t)$ on the brane we thus obtain

$$H_0^2 = \frac{1}{6}\Lambda + \frac{\kappa^4}{36} \left[\sigma + \frac{\rho_*}{(1 - BH_*t)^{2\alpha}} \right]^2. \tag{37}$$

As we shall be mainly interested in the behavior near the Big Rip, we consider times close to the singularity time $t_s = 1/(BH_*)$, where we get approximately

$$\frac{\dot{a}_0}{a_0} = \frac{\kappa^2}{6} \frac{\rho_*}{(1 - BH_*t)^{2\alpha}}. \tag{38}$$

The quantities Λ and σ , characteristic for 5D theory, are here neglected. The solution of this equation is of the form

$$a_0(t) \sim \exp \left[\frac{(\kappa^2/6)\rho_*}{(2\alpha - 1)(BH_*)^{2\alpha}(t_s - t)^{2\alpha-1}} \right]. \tag{39}$$

The scale factor on the brane thus has an essential singularity at $t = t_s$, if $\alpha > 1/2$. This behavior incorporates both Einstein gravity and R^2 -modified gravity ($\alpha = 2$), considered in the previous section. The singularity is stronger, the larger is the value of α . Moreover, the divergence is stronger than the power divergences found for viscous 4D cosmology with account of quantum effects [36]. If $\alpha < 1/2$, a_0 does not diverge at t_s . Note that there is a relationship between ρ_* and H_* in the expression (39), as following from (14) taken at $t = 0$,

$$H_*^2 = \frac{1}{6}\Lambda + \frac{\kappa^4}{36}(\sigma + \rho_*)^2. \tag{40}$$

Now we return to the bulk case, considering (13) for arbitrary y . When $C = 0$, as assumed (recall that also $k = 0$),

we obtain as AdS solution

$$a^2(t, y) = \frac{1}{2}a_0^2(t) \left[\left(1 + \frac{\kappa^4 \sigma^2}{6\Lambda} \right) + \left(1 - \frac{\kappa^4 \sigma^2}{6\Lambda} \right) \cosh(2\mu y) - \frac{\kappa^2 \sigma}{3\mu} \sinh(2\mu|y|) \right], \quad (41)$$

with $\mu = \sqrt{-\Lambda/6}$. This brings us to the following important conclusion: the Big Rip divergence on the brane, present as we have seen when $\alpha > 1/2$, becomes transferred into the bulk. The bulk scale factor $a(t, y)$ diverges for arbitrary y at $t = t_s$, if $a_0(t)$ diverges at t_s . This result could hardly have been seen beforehand, without calculation. There is moreover no particular difference between an Einstein fluid and a modified gravity fluid in this respect; they behave essentially in the same way. It may also be of interest to note that if the brane is tensionless, $\sigma = 0$, then the bulk solution becomes quite simple,

$$a^2(t, y) = \frac{1}{2}a_0^2(t) [1 + \cosh(2\mu y)]. \quad (42)$$

The bulk scale factor thus increases exponentially on both sides of the brane, for large $|y|$.

It would be of physical interest to understand the simultaneous occurrence of singularities on the brane and in the bulk—perhaps there are quantum effects at play here.

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